

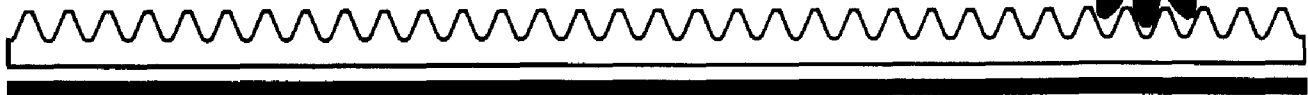
**AGMA 908–B89**  
**(Revision of AGMA 226.01)**

**April 1989**  
**(Reaffirmed August 1999)**

**AMERICAN GEAR MANUFACTURERS ASSOCIATION**

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**Geometry Factors for Determining the Pitting Resistance  
and Bending Strength of Spur, Helical and Herringbone  
Gear Teeth**



**AGMA INFORMATION SHEET**

**(This Information Sheet is not an AGMA Standard)**

## INFORMATION SHEET

### Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical and Herringbone Gear Teeth

AGMA 908–B89

(Revision of AGMA 226.01 1984)

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Suggestions for the improvement of this Standard will be welcome. They should be sent to the American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, Virginia 22314.

### ABSTRACT:

This Information Sheet gives the equations for calculating the pitting resistance geometry factor,  $I$ , for external and internal spur and helical gears, and the bending strength geometry factor,  $J$ , for external spur and helical gears that are generated by rack–type tools (hobs, rack cutters or generating grinding wheels) or pinion–type tools (shaper cutters). The Information Sheet also includes charts which provide geometry factors,  $I$  and  $J$ , for a range of typical gear sets and tooth forms.

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## FOREWORD

[The foreword, footnotes, and appendices are provided for informational purposes only and should not be construed as part of American Gear Manufacturers Association Information Sheet 908–B89, *Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical and Herringbone Gear Teeth.*]

This Information Sheet, AGMA 908–B89, was prepared to assist designers making preliminary design studies, and to present data that might prove useful for those designers without access to computer programs. The tables for geometry factors contained in this Information Sheet do not cover all tooth forms, pressure angles, and pinion and gear modifications, and are not applicable to all gear designs. However, information is also contained for determining geometry factors for other conditions and applications. It is hoped that sufficient geometry factor data is included to be of help to the majority of gear designers.

Geometry factors for strength were first published in Information Sheet AGMA 225.01, March, 1959, *Strength of Spur, Helical, Herringbone and Bevel Gear Teeth.* Additional geometry factors were later published in Standards AGMA 220.02, AGMA 221.02, AGMA 222.02, and AGMA 223.01. AGMA Technical Paper 229.07, October, 1963, *Spur and Helical Gear Geometry Factors*, contained many geometry factors not previously published. Due to the number of requests for this paper, it was decided to publish the data in the form of an Information Sheet which became AGMA 226.01, *Geometry Factors for Determining the Strength of Spur, Helical, Herringbone and Bevel Gear Teeth.*

AGMA 218.01, *AGMA Standard for Rating the Pitting Resistance and Bending Strength of Spur and Helical Involute Gear Teeth*, was published with the methods for determining the geometry factors. When AGMA 218.01 was revised as ANSI/AGMA 2001–B88, the calculation procedures for Geometry Factors,  $I$  and  $J$ , were transferred to this revision of the Geometry Factor Information Sheet. The values of  $I$  and  $J$  factors obtained using the methods of this Information sheet are the same as those of AGMA 218.01. The calculation procedure for  $I$  was simplified, but the end result is mathematically identical. Also, the calculation of  $J$  was modified to include shaper cutters and an equation was added for the addendum modification coefficient,  $x$ , previously undefined and all too often misunderstood. Appendices have been added to document the historical derivation of both  $I$  and  $J$ .

Because an analytical method for calculating the Bending Strength Geometry Factor,  $J$ , is now available, the layout procedure for establishing  $J$  has been eliminated from this document. All references to geometry factors for bevel gears have been removed. This information is now available in AGMA 2003–A86, *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, ZEROL Bevel and Spiral Bevel Gear Teeth.*

The first draft of this Information Sheet, AGMA 908–B89, was presented to the Gear Rating Committee in August, 1987. It was approved by the AGMA Gear Rating Committee on February 24, 1989, after several revisions. It was approved for publication by the AGMA Technical Division Executive Committee on April 21, 1989.

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## 1. Scope

The procedures in this Information Sheet describe the methods for determining Geometry Factors for Pitting Resistance,  $I$ , and Bending Strength,  $J$ . These values are then used in conjunction with the rating procedures described in AGMA 2001-B88, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, for evaluating various spur and helical gear designs produced using a generating process.

**1.1 Pitting Resistance Geometry Factor,  $I$ .** A mathematical procedure is described to determine the Geometry Factor,  $I$ , for internal and external gear sets of spur, conventional helical and low axial contact ratio, LACR, helical designs.

**1.2 Bending Strength Geometry Factor,  $J$ .** A mathematical procedure is described to determine the Geometry Factor,  $J$ , for external gear sets of spur, conventional helical and low axial contact ratio, LACR, helical design. The procedure is valid for generated root fillets, which are produced by both rack and pinion type tools.

**1.3 Tables.** Several tables of precalculated Geometry Factors,  $I$  and  $J$ , are provided for various combinations of gearsets and tooth forms.

**1.4 Exceptions.** The formulas of this Information Sheet are not valid when any of the following conditions exist:

(1) Spur gears with transverse contact ratio less than one,  $m_p < 1.0$ .

(2) Spur or helical gears with transverse contact ratio equal to or greater than two,  $m_p \geq 2.0$ . Additional information on high transverse contact ratio gears is provided in Appendix F.

(3) Interference exists between the tips of teeth and root fillets.

(4) The teeth are pointed.

(5) Backlash is zero.

(6) Undercut exists in an area above the theoretical start of active profile. The effect of this undercut is to move the highest point of single tooth contact, negating the assumption of this calculation method. However, the reduction in tooth root

thickness due to protuberance below the active profile is handled correctly by this method.

(7) The root profiles are stepped or irregular. The  $J$  factor calculation uses the stress correction factors developed by Dolan and Broghamer[1]. These factors may not be valid for root forms which are not smooth curves. For root profiles which are stepped or irregular, other stress correction factors may be more appropriate.

(8) Where root fillets of the gear teeth are produced by a process other than generating.

(9) The helix angle at the standard (reference) diameter\* is greater than 50 degrees.

In addition to these exceptions, the following conditions are assumed:

(a) The friction effect on the direction of force is neglected.

(b) The fillet radius is assumed smooth (it is actually a series of scallops).

**1.5 Bending Stress in Internal Gears.** The Lewis method [2] is an accepted method for calculating the bending stress in external gears, but there has been much research [3] which shows that Lewis' method is not appropriate for internal gears. The Lewis method models the gear tooth as a cantilever beam and is most accurate when applied to slender beams (external gear teeth with low pressure angles), and inaccurate for short, stubby beams (internal gear teeth which are wide at their base). Most industrial internal gears have thin rims, where if bending failure occurs, the fatigue crack runs radially through the rim rather than across the root of the tooth. Because of their thin rims, internal gears have ring-bending stresses which influence both the magnitude and the location of the maximum bending stress. Since the boundary conditions strongly influence the ring-bending stresses, the method by which the internal gear is constrained must be considered. Also, the time history of the bending stress at a particular point on the internal gear is important because the stresses alternate from tension to compression. Because the bending stresses in internal gears are influenced by so many variables, no simplified model for calculating the bending stress in internal gears can be offered at this time.

[ ] Numbers in brackets refer to the bibliography.

\* Refer to AGMA 112.05 for further discussion of standard (reference) diameters.