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# American National Standard

**assessment of  
the assumption of normality  
(employing individual  
observed values)**

N15.15-1974



american national standards institute, inc.  
1430 broadway, new york, new york 10018

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Secretariat

Institute of Nuclear Materials Management

Approved October 3, 1973

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## **National Standard**

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This standard was developed under the procedures of the American National Standards Institute by Subcommittee INMM-3 on Statistics, of Standards Committee N15 on Methods of Nuclear Material Control. The secretariat of N15 is held by the Institute of Nuclear Materials Management.

The Institute of Nuclear Materials Management has recognized the need for standardization of statistical methods and procedures as routinely applied to many activities in the nuclear industry. Many of the statistical procedures are based upon the assumption that the observed data are realizations of normally distributed random variables. It is important that the user of the procedures be aware of whether his normality assumptions are justified.

This standard is intended to describe tests of the assumption of normality for samples for which all of the individual observed values are available. The standard provides information on how to conduct the tests, along with sufficient references for those who wish to delve into their derivations.

Suggestions for improvement of this standard will be welcome. They should be sent to the American National Standards Institute, 1430 Broadway, New York, N.Y. 10018.

This standard was processed and approved for submittal to ANSI by American National Standards Committee on Methods of Nuclear Material Control, N15. Committee approval of the standard does not necessarily imply that all committee members voted for its approval. At the time it approved this standard, the N15 Committee had the following members:

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# Assessment of the Assumption of Normality (Employing Individual Observed Values)

## 1. Scope and Purpose

This standard describes statistical hypothesis-testing techniques that are designed to assist in the assessment of the assumption of normality by making use of all the individual observed values in a sample. There is no requirement that the number of observed values be "large," nor is it required that the observations be grouped into intervals with the attendant loss of information about the interval spacings. The procedures are called "omnibus" because they are appropriate for detecting departures from normality caused by any of several characteristics, including skewness (nonsymmetry) and kurtosis ("peakedness"). Empirical studies have demonstrated that these procedures generally are more effective than several others in wide currency (further details appear in Appendix A).

## 2. Background

2.1 The two statistical hypothesis-testing techniques described in this standard (the  $W$  and  $D'$  tests) are based on the same principle: comparison of a "linear combination" estimator of the population variance with the conventional "sum of squared deviations" estimator of the population variance. However, the details of the computation and the appropriate critical values depend on whether the number of observations is less than or equal to 50 or greater than 50.

2.1.1 When the number of observations is "small" (that is, when the sample size is  $n \leq 50$ ), the technique to be used is known as the  $W$  test, which evaluates the assumption of a normal distribution. The  $W$  test was developed by Shapiro and Wilk [1]\* and later popularized by Hahn and Shapiro (see pp 294–298 of Reference [2]).

2.1.2 When the number of observations is "not small" (that is, when the sample size is  $n > 50$ ), the technique to be used is known as the  $D'$  ( $D$  prime) test

of normality for moderate and large size samples. The  $D'$  test is based on a development by D'Agostino [3].

2.2 Both the  $W$  and the  $D'$  tests are applicable to either "raw" or "transformed" observations. Raw observations are values obtained directly from an experiment; transformed observations are those values derived from raw observations by subjecting them to numerical or mathematical manipulation (such as finding their logarithms).

Some remarks justifying the selection of the  $W$  and  $D'$  tests for this standard are given in Appendix A.

## 3. Statistical Considerations

3.1 Application of this standard is enhanced by the user's familiarity with certain statistical terminology. The concepts of "hypothesis testing" and "normally distributed variables" are particularly important.

3.2 Statistical hypothesis testing consists of procedural and operational steps.

3.2.1 A statistical test of a hypothesis includes, according to Dixon and Massey (see p 82 of Reference [4]), the following steps:

- (1) Statement of hypothesis and assumptions.
- (2) Choice of level of significance. (Usually, the level of significance is denoted by the Greek letter  $\alpha$  and consists of a small value such as 0.01 or 0.05; sometimes it is referred to as the probability of committing an error of the first kind.)
- (3) Determination of a test statistic and a critical region.
- (4) Computation and display of appropriate statistics.
- (5) Full statement of conclusions.

3.2.2 The operational details followed in this standard are summarized by Hahn and Shapiro (see pp 294–295 of Reference [2]) in three basic steps:

- (1) The test statistic is calculated from the observed data.
- (2) The probability of obtaining the calculated test

\*Numbers in brackets refer to corresponding numbers in Section 9, References to the Text.