



ISO 21771-1

Cylindrical involute gears and gear pairs —

Part 1:
Concepts and geometry

*Roues et engrenages cylindriques en développante de cercle —
Partie 1: Concepts et géométrie*

**First edition
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Foreword	vii
1 Scope	1
2 Normative references	1
3 Terms, definitions, symbols, subscripts and units	1
3.1 Terms and definitions.....	1
3.2 Symbols.....	2
3.3 Subscripts.....	9
3.4 Units and conventions.....	10
3.4.1 Units.....	10
3.4.2 Conventions.....	10
4 Individual cylindrical gears	10
4.1 General.....	10
4.2 Concepts for an individual gear.....	11
4.2.1 Gear, cylindrical gear, external gear and internal gear.....	11
4.2.2 Tooth system, external teeth and internal teeth.....	11
4.2.3 Tooth and tooth space.....	11
4.2.4 Tooth system parameters.....	11
4.2.5 Number of teeth and sign of number of teeth.....	11
4.2.6 Tooth number.....	12
4.2.7 Top land and bottom land.....	12
4.2.8 Tooth flanks and flank sections.....	12
4.3 Reference surfaces, datum lines and reference quantities.....	13
4.3.1 Reference surface and datum face.....	13
4.3.2 Reference rack.....	13
4.3.3 Basic rack tooth profile for involute gear teeth.....	14
4.3.4 Reference cylinder, reference circle and reference diameter.....	15
4.3.5 Gear axis.....	15
4.3.6 Sections through a cylindrical gear.....	15
4.3.7 Module.....	16
4.3.8 Diametral pitch.....	16
4.3.9 Facewidth.....	16
4.3.10 Profile shift, profile shift coefficient and sign of profile shift.....	17
4.4 Involute helicoids.....	18
4.4.1 Generator of involute helicoids.....	18
4.4.2 Base cylinder, base circle and base diameter.....	19
4.4.3 Transverse pressure angle at a point and transverse pressure angle.....	19
4.4.4 Normal pressure angle at a point and normal pressure angle.....	20
4.4.5 Involute function.....	20
4.4.6 Lead.....	21
4.4.7 Helix angle and lead angle.....	21
4.4.8 Hand of helix.....	22
4.4.9 Roll angle of the involute.....	23
4.4.10 Radius of curvature of the involute and length of roll.....	23
4.4.11 Length of involute profile, L_{yt}	23
4.5 Angular pitch and pitches.....	23
4.5.1 Angular pitch.....	23
4.5.2 Pitches on the reference cylinder.....	23
4.5.3 Pitches on any cylinder.....	24
4.5.4 Axial pitch.....	25
4.5.5 Base pitch.....	25
4.6 Diameters of gear teeth.....	25
4.6.1 General.....	25
4.6.2 V-cylinder and V-circle diameter.....	26
4.6.3 y-cylinder and y-circle diameter.....	26

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	4.6.6	Profile control diameter, d_{cf}	26
	4.6.7	Root cylinder, root circle and root diameter.....	26
4.7		Gear tooth depth.....	27
	4.7.1	Tooth depth.....	27
	4.7.2	Addendum, working addendum, dedendum and working dedendum.....	27
4.8		Tooth thickness and space width.....	27
	4.8.1	General.....	27
	4.8.2	Transverse tooth thickness.....	27
	4.8.3	Transverse tooth thickness half angle.....	28
	4.8.4	Transverse space width.....	29
	4.8.5	Transverse space width half angle.....	29
	4.8.6	Normal tooth thickness.....	29
	4.8.7	Normal space width.....	29
4.9		Radii of curvature of the tooth flanks in different planes.....	30
4.10		Tilt angle of contact line.....	31
5		Parallel axis cylindrical gear pairs.....	32
	5.1	General.....	32
	5.2	Concepts for a gear pair.....	32
	5.2.1	Mating gear and mating flank.....	32
	5.2.2	Working flank and non-working flank.....	32
	5.2.3	External gear pair.....	32
	5.2.4	Internal gear pair.....	33
	5.3	Mating quantities.....	33
	5.3.1	Gear ratio.....	33
	5.3.2	Driving gear, driven gear and transmission ratio.....	33
	5.3.3	Line of centres and centre distance.....	33
	5.3.4	Working transverse pressure angle.....	34
	5.3.5	Working pitch point, pitch cylinders, pitch circles, pitch diameter and pitch axis.....	34
	5.3.6	Working helix angles.....	36
	5.3.7	Working depth.....	36
	5.3.8	Tip clearance.....	36
	5.3.9	Calculation of tip alteration coefficient k (for parallel axis gear).....	37
	5.4	Calculation of the sum of the profile shift coefficients.....	38
	5.5	Tooth engagement.....	38
	5.5.1	General.....	38
	5.5.2	Active area of the tooth flanks, start of active profile and active tip diameters.....	38
	5.5.3	Plane of action, zone of action and contact line.....	40
	5.5.4	Line of action, path of contact and point of contact.....	41
	5.5.5	Form over dimension.....	41
	5.5.6	Designations and values relating to the line of action.....	41
	5.5.7	Radii of curvature of the tooth flanks in a transverse plane.....	42
	5.5.8	Tooth interference.....	43
	5.5.9	Mesh geometry parameters.....	45
	5.5.10	Contact line and sum of the contact line lengths.....	47
	5.6	Backlash.....	49
	5.6.1	General.....	49
	5.6.2	Transverse backlash.....	49
	5.6.3	Circumferential backlash.....	49
	5.6.4	Normal backlash.....	49
	5.6.5	Radial backlash.....	50
	5.6.6	Angular backlash.....	50
	5.7	Velocities and sliding conditions at the tooth flanks.....	51
	5.7.1	Angular velocity.....	51
	5.7.2	Circumferential velocity.....	51
	5.7.3	Normal velocity.....	51
	5.7.4	Rolling velocity.....	52

	5.7.7	Specific sliding.....	54
6		Crossed axis cylindrical gear pairs.....	54
	6.1	General.....	54
	6.2	Concepts for a gear pair.....	55
	6.3	Mating quantities.....	56
	6.3.1	Gear ratio.....	56
	6.3.2	Driving gear, driven gear and transmission ratio.....	56
	6.3.3	Working pitch surfaces and working pitch point.....	56
	6.3.4	Line of centres and working centre distance.....	56
	6.3.5	Skew angle.....	57
	6.3.6	Intermeshing racks at working pitch cylinders.....	57
	6.3.7	Working transverse pressure angles.....	59
	6.3.8	Working transverse tooth thicknesses.....	59
	6.3.9	Basic rack profile to generate each part of the gear pair.....	60
	6.3.10	Profile shift coefficients.....	60
	6.3.11	Base space width half angle.....	61
	6.3.12	Working conditions at minimum centre distance.....	61
	6.3.13	Working conditions with backlash.....	62
	6.3.14	Working depth.....	62
	6.3.15	Tip clearance.....	62
	6.3.16	Calculation of tip alteration coefficient k (for crossed axis gear).....	62
	6.4	Tooth engagement.....	62
	6.4.1	General.....	62
	6.4.2	Line of action, path of contact and point of contact.....	63
	6.4.3	Active area of the tooth flanks, start of active profile and active tip diameters.....	64
	6.4.4	Form over dimension.....	65
	6.4.5	Designations and values relating to the line of action.....	65
	6.4.6	Tooth interference.....	66
	6.4.7	Mesh geometry parameters.....	67
	6.5	Backlash.....	67
	6.5.1	General.....	67
	6.5.2	Normal base backlash.....	68
	6.5.3	Backlash angle and circumferential backlash.....	68
	6.6	Sliding conditions at the tooth flanks.....	69
	6.6.1	General.....	69
	6.6.2	Angular velocities vectors.....	69
	6.6.3	Velocity vectors of a point of contact along path of contact.....	70
	6.6.4	Sliding velocity.....	71
7		Principal radii of curvature of the tooth flanks.....	71
	7.1	General.....	71
	7.2	Angle between the principal radius of curvature at point of contact.....	72
	7.3	Particularities for crossed axis helical involute gear.....	73
	7.4	Equivalent radii of curvature at point of contact.....	74
8		Tooth flank modifications.....	75
	8.1	General.....	75
	8.2	Tooth modifications which restrict the usable flank.....	75
	8.2.1	Pre-finish flank undercut.....	75
	8.2.2	Tip corner chamfering, tip corner rounding.....	75
	8.3	Transverse profile modifications.....	77
	8.3.1	General.....	77
	8.3.2	Tip and root relief, $C_{\alpha a}$ $C_{\alpha f}$	77
	8.3.3	Transverse profile slope modification, $C_{H\alpha}$	77
	8.3.4	Profile crowning (barrelling), C_{α}	78
	8.4	Flank line (helix) modifications.....	79
	8.4.1	Flank line end relief, $C_{H\beta}$	79

This is a preview of ISO 21771-1:2024. [Click here to purchase the full version from the ANSI store.](#)

8.5	Flank face modifications.....	80
8.5.1	Topographical modifications.....	80
8.5.2	Triangular end relief.....	81
8.5.3	Flank twist.....	82
8.6	Descriptions of modifications by functions.....	82
9	Geometrical limits.....	83
9.1	General.....	83
9.2	Counterpart rack tooth profile and rack tool profile.....	84
9.2.1	Counterpart rack tooth profile.....	84
9.2.2	Rack tool profile.....	87
9.3	Machining allowance.....	91
9.4	Limits of normal tooth thickness.....	91
9.5	Generating profile shift, generating profile shift coefficient.....	92
9.6	Generated root diameter.....	93
9.7	Usable area of the tooth flank, tip and root form diameter.....	93
9.8	Undercut.....	95
9.9	Overcut.....	96
9.10	Minimum tooth thickness at the tip circle of a gear.....	96
10	Start of involute for hob or rack type cutters.....	96
10.1	Formulae of involute and trochoid.....	96
10.2	Undercut conditions.....	98
10.3	Determination of start of involute.....	99
10.4	Determination of radius of curvature of trochoid.....	101
11	Start of involute calculation in case of pinion-type cutter.....	103
11.1	Geometry of pinion-type cutter.....	103
11.1.1	General.....	103
11.1.2	Pinon-type cutter without protuberance.....	105
11.1.3	Pinon-type cutter with protuberance.....	106
11.2	Undercut condition.....	107
11.3	Determination of start of involute when no undercut exists.....	108
11.4	Start of involute when there is undercut.....	108
11.4.1	Residual fillet undercut (transverse plane).....	108
11.4.2	Angle centre line of gear pair to centre point of tooth tip rounding:.....	108
11.4.3	Point Q (tangent point of cutter tip fillet and generated gear root fillet).....	109
11.4.4	Determination of start of involute.....	109
11.4.5	Determination of radius of curvature of trochoid.....	112
Annex A	(informative) Form diameters when a circular arc in a transverse plane is used for tooth tip corner radius or tooth root fillet radius.....	114
Annex B	(informative) Tip form radius when tooth tip corner radius is defined on normal surface with pinion type cutter.....	123
Annex C	(informative) Supplementary information for crossed axes cylindrical gears.....	131
Annex D	(informative) Geometry of surfaces in contact.....	138
Annex E	(informative) Projection of transverse profile of the tooth to other planes.....	148
Annex F	(informative) Interface for geometry of involute worms defined with ISO 10828.....	156
Bibliography	159

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This document was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 1, *Nomenclature and wormgearing*.

This first edition of ISO 21771-1 cancels and replaces ISO 21771:2007, which has been technically revised.

The main changes are as follows:

- the sign convention for internal gears used in the ISO 6336 series^[8] has been adopted. The negative value for the number of teeth of an internal gear is applied to the diameters and centre distance, so these dimensions of internal gears have negative values;
- flank direction has been renamed as hand of helix and sign (+/-) of helix angle is used;
- a definition of normal surface has been added and this is used rather than normal plane;
- the annex on tooth thickness was removed because it is now addressed in ISO 21771-2.

Additional material has been added to cover:

- calculation of form diameters for tooth tip corner radius and tooth root fillet radius in the transverse plane for an involute cylindrical gear ([Clauses 10, 11](#) and [Annex B](#));
- calculation of the tooth tip corner radius for a specified form diameter and tip diameter of an involute cylindrical gear;
- calculation of a radius tangent to the involutes of adjacent teeth at root or tip diameter ([Annex A](#));
- generated tooth root fillet shape for individual involute cylindrical gears ([Annex B](#));
- concepts and parameters for involute cylindrical gear pairs with crossed axes ([Clause 6](#) and [Annex C](#));
- geometry of surfaces in contact ([Annex D](#));
- projection of a transverse plane profile of a tooth onto another plane ([Annex E](#));

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