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# Control charts —

## Part 7: Multivariate control charts

*Cartes de contrôle —*

*Partie 7: Cartes de contrôle multivariées*



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## Contents

|   | Page      |
|---|-----------|
| <b>Foreword</b> .....   | <b>iv</b> |
| <b>Introduction</b> .....   | <b>v</b>  |
| <b>1 Scope</b> .....  | <b>1</b>  |
| <b>2 Normative references</b> .....   | <b>1</b>  |
| <b>3 Terms and definitions</b> .....  | <b>1</b>  |
| <b>4 Abbreviated terms and symbols</b> .....  | <b>2</b>  |
| 4.1 Abbreviated terms.....  | 2         |
| 4.2 Symbols.....  | 2         |
| <b>5 Purpose and classification of multivariate control charts</b> .....                                  | <b>4</b>  |
| 5.1 Purpose and applying conditions for multivariate control charts.....                                  | 4         |
| 5.2 Classification of multivariate control charts.....  | 5         |
| <b>6 Multivariate control charts with unweighted averages for mean shift</b> .....                        | <b>6</b>  |
| 6.1 General.....  | 6         |
| 6.2 Control charts for the process mean ( $n > 1$ ).....  | 7         |
| 6.2.1 $\chi^2$ control chart when pre-specified parameter values are known.....                           | 7         |
| 6.2.2 $T^2$ control chart when pre-specified parameter values are unknown.....                            | 8         |
| 6.3 Control charts for the process mean ( $n = 1$ ).....  | 8         |
| 6.3.1 $\chi^2$ control chart when pre-specified parameter values are known.....                           | 8         |
| 6.3.2 $T^2$ control chart when pre-specified parameter values are unknown.....                            | 9         |
| 6.4 Summary and selection of multivariate control charts with unweighted averages<br>for mean shifts..... | 9         |
| 6.5 Test for assignable causes.....   | 10        |
| <b>7 Multivariate control charts with weighted averages for mean shifts</b> .....                         | <b>11</b> |
| <b>8 Control charts for the process dispersion</b> .....  | <b>12</b> |
| <b>9 Interpretation of an out-of-control signal</b> .....   | <b>13</b> |
| <b>Annex A (informative) Example of multivariate statistical process control</b> .....                    | <b>14</b> |
| <b>Annex B (informative) Example of MEWMA control chart</b> .....   | <b>17</b> |
| <b>Annex C (informative) Estimation of <math>\mu</math> and <math>\Sigma</math></b> .....                 | <b>23</b> |
| <b>Bibliography</b> .....   | <b>25</b> |

## Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see [www.iso.org/directives](http://www.iso.org/directives)).

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This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

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## Introduction

When a number of quality characteristics are to be controlled simultaneously, the usual practice has been to maintain a separate (univariate) chart for each characteristic. Unfortunately, this can give misleading results when the characteristics are highly correlated. Process monitoring of problems in which several related variables are of interest are collectively known as multivariate statistical process control (MSPC). The most useful tools of multivariate statistical process control are multivariate control charts. Multivariate control charts are applied for statistical process evaluation and control under the consideration of dependability between quality characteristics.

The function of a multivariate statistical process control system is to provide a statistical signal when assignable causes of variation are present. The systematic elimination of assignable causes of excessive variation, through continuous determined efforts, brings the process into a state of statistical control. Once the process is operating in statistical control, its performance is predictable and its capability to meet the specifications can then be assessed.

The main purpose of this document is to show how multivariate control charts can be used for process control in terms of SPC and how the state of process stability can be assessed in a multivariate way. ISO 22514-6 provides a calculation method for capability statistics for process parameters or product characteristics following a multivariate normal distribution or approximately multivariate normal.

Multivariate charts are based on multivariate characteristics where more than one characteristic is to be monitored in connection with others. In practice, a multivariate control chart is always applied with the support of software, such as Minitab, JMP, and Q-DAS<sup>1)</sup>.

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1) MINITAB is the trade name of a product supplied by Minitab Inc. JMP is the trade name of a product supplied by SAS Institute Inc. Q-DAS is the trade name of a product supplied by Q-DAS GmbH. This information is given for the convenience of users of this document and does not constitute an endorsement by ISO of these products.