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Representation of results of particle size analysis —

Part 3: Adjustment of an experimental curve to a reference model

Représentation de données obtenues par analyse granulométrique —

Partie 3: Ajustement d'une courbe expérimentale à un modèle de référence

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

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The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

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ISO 9276-3 was prepared by Technical Committee ISO/TC 24, *Sieves, sieving and other sizing methods*, Subcommittee SC 4, *Sizing by methods other than sieving*.

ISO 9276 consists of the following parts, under the general title *Representation of results of particle size analysis*:

— *Part 1: Graphical representation*

— *Part 2: Calculation of average particle sizes/diameters and moments from particle size distributions*

— *Part 3: Adjustment of an experimental curve to a reference model*

— *Part 4: Characterization of a classification process*

— *Part 5: Methods of calculation relating to particle size analyses using logarithmic normal probability distribution*

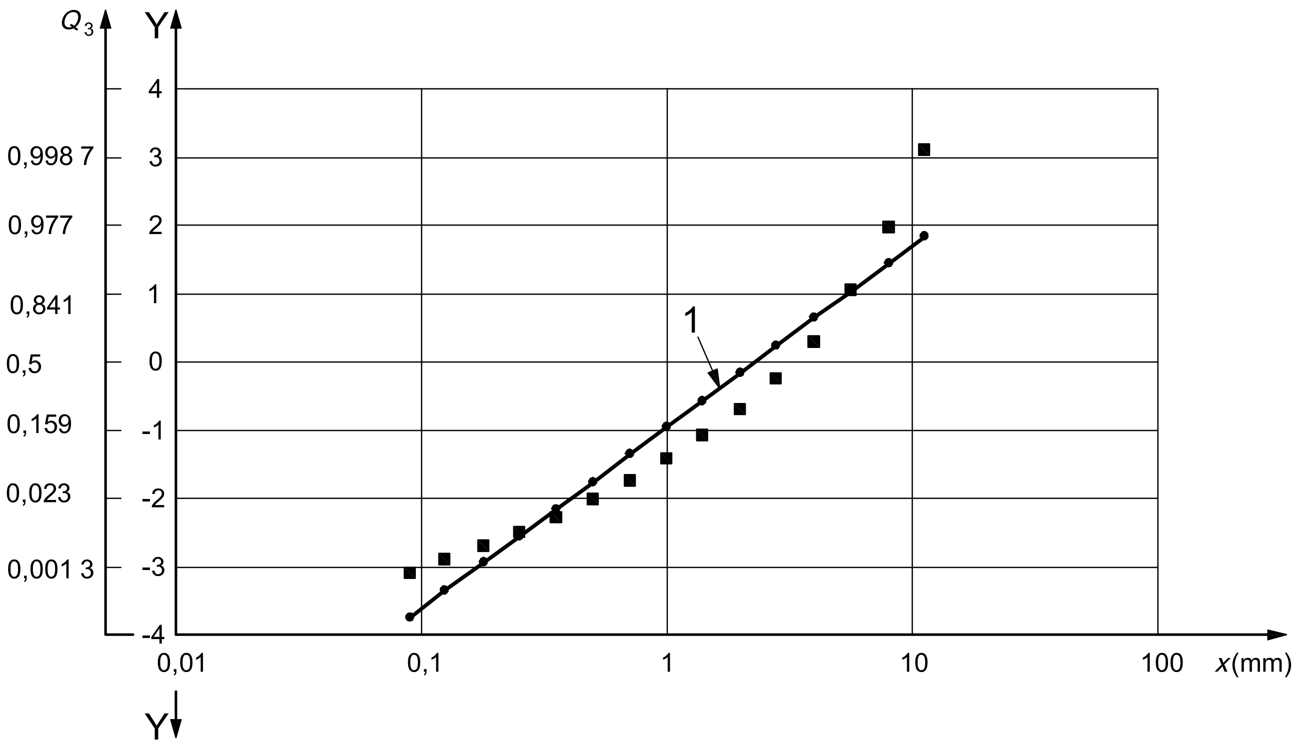
The following part is under preparation:

— *Part 6: Descriptive and quantitative representation of particle shape and morphology*

Introduction

Cumulative curves of particle size distributions are sigmoids, therefore fitting to a model distribution function or rendering statistical intercomparison is difficult. These disadvantages can, however, be remedied by transforming these sigmoids into straight lines by means of appropriate coordinate systems, e.g. log-normal, Rosin-Rammler or Gates-Gaudin-Schuhmann (log-log). Target size distributions in particle technology industries can also be described in terms of distribution models.

In such systems, a classic linear regression assumes that the squares of the deviations between the experimental points and the theoretical straight line are, on average, equal. This is only valid in the transformed cumulative distribution value system, but not in their linear representation, and therefore named a quasilinear regression. In particular, the scale extension makes the values of the squares of the deviations at the extremities of the graph vary by several orders of magnitude. In addition, the sum of the squares of the deviations obtained by this method is not related to any simple distribution and does not allow any statistical test.



Key

$Q_3(x)$ cumulative distribution by volume or mass

x particle size

Y quantiles of the standard normal distribution

1 quasilinear regression full line

• quasilinear fit point

▪ $Q_3(x)$ data point

Figure 1 — Example of a functional paper with log-normal plot (cumulative distribution values plotted on a normal ordinate against particle size on a logarithmic abscissa with inverse standard normal distribution transformed) and quasilinear regression full line

The experimental data in Figure 1 are taken from ISO 9276-1:1998[1], Annex A and represent a sieve-measuring result example between 90 µm and 11,2 mm.

The mathematical treatment, corresponding to non-linear coordinate systems, mentioned above, agrees with a quasilinear regression. Here the non-linear transformation of the Y-axis results in a non-linear transformation of the Y-deviations, e.g. another consideration of deviations at the tails of a distribution than at their centre.

One possibility to compensate for the non-linear transformation of the Y-differences, in the result of the non-linear transformation of the Y values, is the introduction of weighting factors in the quasilinear regression (see Annex E).

Moreover, a non-linear regression delivers the best adjustment and allows the most flexibility, such as statistical tests on number distributions, the adjustment of truncated or multimodal distributions or any other arbitrary models, but it requires a start approximation and a numerical mathematical procedure.

The standard deviation of residuals between experimental points and the model in the non-transformed scale allows the quantification of the degree of alignment and the statistical comparison of experimental distributions. A value of greater than e.g. 0,05 indicates a non-adequate reference model.

