



GUIDE 98-3/Suppl.2

Uncertainty of measurement —

Part 3:

**Guide to the expression of
uncertainty in measurement
(GUM:1995)**

Supplement 2:

**Extension to any number of
output quantities**

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ISO/IEC Guide 98-3/Suppl.2 was prepared by Working Group 1 of the Joint Committee for Guides in Metrology (as JCGM 102:2011), and was adopted by the national bodies of ISO and IEC.

ISO/IEC Guide 98 consists of the following parts, under the general title *Uncertainty of measurement*:

- *Part 1: Introduction to the expression of uncertainty in measurement*
- *Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*
- *Part 4: Role of measurement uncertainty in conformity assessment*

The following parts are planned:

- *Part 2: Concepts and basic principles*
- *Part 5: Applications of the least-squares method*

ISO/IEC Guide 98-3 has three supplements:

- *Supplement 1: Propagation of distributions using a Monte Carlo method*
- *Supplement 2: Models with any number of output quantities*
- *Supplement 3: Modelling*

Given that ISO/IEC Guide 98-3:2008/Suppl.2:2011 is identical in content to JCGM 102:2011, the decimal symbol is a point on the line in the English version.

Annex ZZ has been appended to provide a list of corresponding ISO/IEC Guides and JCGM guidance documents for which equivalents are not given in the text.

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2011

Evaluation of measurement data — Supplement 2 to the “Guide to the expression of uncertainty in measurement” — Extension to any number of output quantities

Évaluation des données de mesure — Supplément 2 du “Guide pour l’expression
de l’incertitude de mesure” — Extension à un nombre quelconque de grandeurs
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FOREWORD

In 1997 a Joint Committee for Guides in Metrology (JCGM), chaired by the Director of the Bureau International des Poids et Mesures (BIPM), was created by the seven international organizations that had originally in 1993 prepared the “Guide to the expression of uncertainty in measurement” (GUM) and the “International vocabulary of basic and general terms in metrology” (VIM). The JCGM assumed responsibility for these two documents from the ISO Technical Advisory Group 4 (TAG4).

The Joint Committee is formed by the BIPM with the International Electrotechnical Commission (IEC), the International Federation of Clinical Chemistry and Laboratory Medicine (IFCC), the International Laboratory Accreditation Cooperation (ILAC), the International Organization for Standardization (ISO), the International Union of Pure and Applied Chemistry (IUPAC), the International Union of Pure and Applied Physics (IUPAP), and the International Organization of Legal Metrology (OIML).

JCGM has two Working Groups. Working Group 1, “Expression of uncertainty in measurement”, has the task to promote the use of the GUM and to prepare Supplements and other documents for its broad application. Working Group 2, “Working Group on International vocabulary of basic and general terms in metrology (VIM)”, has the task to revise and promote the use of the VIM.

Supplements such as this one are intended to give added value to the GUM by providing guidance on aspects of uncertainty evaluation that are not explicitly treated in the GUM. The guidance will, however, be as consistent as possible with the general probabilistic basis of the GUM.

The present Supplement 2 to the GUM has been prepared by Working Group 1 of the JCGM, and has benefited from detailed reviews undertaken by member organizations of the JCGM and National Metrology Institutes.

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INTRODUCTION

The “Guide to the expression of uncertainty in measurement” (GUM) [JCGM 100:2008] is mainly concerned with univariate measurement models, namely models having a single scalar output quantity. However, models with more than one output quantity arise across metrology. The GUM includes examples, from electrical metrology, with three output quantities [JCGM 100:2008 H.2], and thermal metrology, with two output quantities [JCGM 100:2008 H.3]. This Supplement to the GUM treats multivariate measurement models, namely models with any number of output quantities. Such quantities are generally mutually correlated because they depend on common input quantities. A generalization of the GUM uncertainty framework [JCGM 100:2008 5] is used to provide estimates of the output quantities, the standard uncertainties associated with the estimates, and covariances associated with pairs of estimates. The input or output quantities in the measurement model may be real or complex.

Supplement 1 to the GUM [JCGM 101:2008] is concerned with the propagation of probability distributions [JCGM 101:2008 5] through a measurement model as a basis for the evaluation of measurement uncertainty, and its implementation by a Monte Carlo method [JCGM 101:2008 7]. Like the GUM, it is only concerned with models having a single scalar output quantity [JCGM 101:2008 1]. This Supplement describes a generalization of that Monte Carlo method to obtain a discrete representation of the joint probability distribution for the output quantities of a multivariate model. The discrete representation is then used to provide estimates of the output quantities, and standard uncertainties and covariances associated with those estimates. Appropriate use of the Monte Carlo method would be expected to provide valid results when the applicability of the GUM uncertainty framework is questionable, namely when (a) linearization of the model provides an inadequate representation, or (b) the probability distribution for the output quantity (or quantities) departs appreciably from a (multivariate) Gaussian distribution.

Guidance is also given on the determination of a coverage region for the output quantities of a multivariate model, the counterpart of a coverage interval for a single scalar output quantity, corresponding to a stipulated coverage probability. The guidance includes the provision of coverage regions that take the form of hyper-ellipsoids and hyper-rectangles. A calculation procedure that uses results provided by the Monte Carlo method is also described for obtaining an approximation to the smallest coverage region.