

This is a preview of "ISO/TR 13587:2012". [Click here to purchase the full version from the ANSI store.](#)

First edition  
2012-07-15

---

---

## Three statistical approaches for the assessment and interpretation of measurement uncertainty

*Trois approches statistiques pour l'évaluation et l'interprétation de  
l'incertitude de mesure*



Reference number  
ISO/TR 13587:2012(E)

© ISO 2012

This is a preview of "ISO/TR 13587:2012". [Click here to purchase the full version from the ANSI store.](#)



**COPYRIGHT PROTECTED DOCUMENT**

© ISO 2012

All rights reserved. Unless otherwise specified, no part of this publication may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying and microfilm, without permission in writing from either ISO at the address below or ISO's member body in the country of the requester.

ISO copyright office  
Case postale 56 • CH-1211 Geneva 20  
Tel. + 41 22 749 01 11  
Fax + 41 22 749 09 47  
E-mail [copyright@iso.org](mailto:copyright@iso.org)  
Web [www.iso.org](http://www.iso.org)

Published in Switzerland

This is a preview of "ISO/TR 13587:2012". [Click here to purchase the full version from the ANSI store.](#)

## Contents

Page

Foreword .....	v
Introduction.....	vi
1 Scope.....	1
2 Normative references.....	1
3 Terms and definitions .....	1
4 Symbols (and abbreviated terms).....	2
5 The problem addressed.....	3
6 Statistical approaches .....	4
6.1 Frequentist approach.....	4
6.2 Bayesian approach.....	5
6.3 Fiducial approach.....	5
6.4 Discussion .....	6
7 Examples.....	6
7.1 General .....	6
7.2 Example 1a.....	6
7.3 Example 1b.....	7
7.4 Example 1c.....	7
8 Frequentist approach to uncertainty evaluation.....	7
8.1 Basic method .....	7
8.2 Bootstrap uncertainty intervals .....	10
8.3 Example 1 .....	13
8.3.1 General .....	13
8.3.2 Example 1a.....	14
8.3.3 Example 1b.....	15
8.3.4 Example 1c.....	15
9 Bayesian approach for uncertainty evaluation .....	16
9.1 Basic method .....	16
9.2 Example 1 .....	18
9.2.1 General .....	18
9.2.2 Example 1a.....	18
9.2.3 Example 1b.....	20
9.2.4 Example 1c.....	21
9.2.5 Summary of example .....	21
10 Fiducial inference for uncertainty evaluation.....	21
10.1 Basic method .....	21
10.2 Example 1 .....	23
10.2.1 Example 1a.....	23
10.2.2 Example 1b.....	25
10.2.3 Example 1c.....	26
11 Example 2: calibration of a gauge block.....	26
11.1 General .....	26
11.2 Frequentist approach.....	28
11.3 Bayesian approach.....	30
11.4 Fiducial approach.....	33
12 Discussion .....	35
12.1 Comparison of uncertainty evaluations using the three statistical approaches.....	35

This is a preview of "ISO/TR 13587:2012". [Click here to purchase the full version from the ANSI store.](#)

12.2	Relation between the methods proposed in GUM Supplement 1 (GUMS1) and the three statistical approaches .....	38
13	Summary .....	40
	Bibliography .....	42

This is a preview of "ISO/TR 13587:2012". [Click here to purchase the full version from the ANSI store.](#)

## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In exceptional circumstances, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example), it may decide by a simple majority vote of its participating members to publish a Technical Report. A Technical Report is entirely informative in nature and does not have to be reviewed until the data it provides are considered to be no longer valid or useful.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TR 13587:2012 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 6, *Measurement methods and results*.

This Technical Report is primarily based on Reference [10].

## Introduction

The adoption of ISO/IEC Guide 98-3 (GUM) <sup>[1]</sup> has led to an increasing recognition of the need to include uncertainty statements in measurement results. Laboratory accreditation based on International Standards like ISO 17025 <sup>[2]</sup> has accelerated this process. Recognizing that uncertainty statements are required for effective decision-making, metrologists in laboratories of all types, from National Metrology Institutes to commercial calibration laboratories, are exerting considerable effort on the development of appropriate uncertainty evaluations for different types of measurement using methods given in the GUM.

Some of the strengths of the procedures outlined and popularized in the GUM are its standardized approach to uncertainty evaluation, its accommodation of sources of uncertainty that are evaluated either statistically (Type A) or non-statistically (Type B), and its emphasis on reporting all sources of uncertainty considered. The main approach to uncertainty propagation in the GUM, based on linear approximation of the measurement function, is generally simple to carry out and in many practical situations gives results that are similar to those obtained more formally. In short, since its adoption, the GUM has sparked a revolution in uncertainty evaluation.

Of course, there will always be more work needed to improve the evaluation of uncertainty in particular applications and to extend it to cover additional areas. Among such other work, the Joint Committee for Guides in Metrology (JCGM), responsible for the GUM since the year 2000, has completed Supplement 1 to the GUM, namely, "Propagation of distributions using a Monte Carlo method" (referred to as GUMS1) <sup>[3]</sup>. The JCGM is developing other supplements to the GUM on topics such as modelling and models with any number of output quantities.

Because it should apply to the widest possible set of measurement problems, the definition of measurement uncertainty in ISO/IEC Guide 99:2007 <sup>[4]</sup> as a "non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used" cannot reasonably be given at more than a relatively conceptual level. As a result, defining and understanding the appropriate roles of different statistical quantities in uncertainty evaluation, even for relatively well-understood measurement applications, is a topic of particular interest to both statisticians and metrologists.

Earlier investigations have approached these topics from a metrological point of view, some authors focusing on characterizing statistical properties of the procedures given in the GUM. Reference [5] shows that these procedures are not strictly consistent with either a Bayesian or frequentist interpretation. Reference [6] proposes some minor modifications to the GUM procedures that bring the results into closer agreement with a Bayesian interpretation in some situations. Reference [7] discusses the relationship between procedures for uncertainty evaluation proposed in GUMS1 and the results of a Bayesian analysis for a particular class of models. Reference [8] also discusses different possible probabilistic interpretations of coverage intervals and recommends approximating the posterior distributions for this class of Bayesian analyses by probability distributions from the Pearson family of distributions.

Reference [9] compares frequentist ("conventional") and Bayesian approaches to uncertainty evaluation. However, the study is limited to measurement systems for which all sources of uncertainty can be evaluated using Type A methods. In contrast, measurement systems with sources of uncertainty evaluated using both Type A and Type B methods are treated in this Technical Report and are illustrated using several examples, including one of the examples from Annex H of the GUM.

Statisticians have historically placed strong emphasis on using methods for uncertainty evaluation that have probabilistic justification or interpretation. Through their work, often outside metrology, several different approaches for statistical inference relevant to uncertainty evaluation have been developed. This Technical Report presents some of those approaches to uncertainty evaluation from a statistical point of view and relates them to the methods that are currently being used in metrology or are being developed within the metrology community. The particular statistical approaches under which different methods for uncertainty evaluation will be described are the frequentist, Bayesian, and fiducial approaches, which are discussed further after outlining the notational conventions needed to distinguish different types of quantities.